

TWIN AND MIRROR SYMMETRIES FROM PRESYMMETRY

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We argue that presymmetry, a hidden predynamical electroweak quark–lepton symmetry that explains the fractional charges and triplication of families, must be extended beyond the Standard Model as to have a residual presymmetry that embraces partner particles and includes the strong sector, so accounting for the twin or mirror partners proposed to alleviate the naturalness problem of the weak scale. It leads to the full duplication of fermions and gauge bosons of the Standard Model independently of the ultraviolet completion of the theory, even if the Higgs particle is discarded by experiment, which adds robustness to twin and mirror symmetries. The established connection is so strongly motivated that the search for twin or mirror matter becomes the possible test of presymmetry. If the physics beyond the Standard Model repairs its left–right asymmetry, mirror symmetry should be the one realized in nature.

Keywords: Charge symmetries; presymmetry; topological charge; twin symmetry; mirror symmetry; hierarchy problem.

PACS numbers: 11.30.Hv, 11.30.Ly, 12.60.Fr, 12.60.Cn

1. Introduction

The Standard Model (SM) of strong and electroweak interactions with gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$ and extended with right-handed neutrinos has a remarkable success in explaining all experimental data obtained so far. To be completely accepted, however, the predicted neutral Higgs boson has to be discovered. Electroweak high precision measurements imply a light Higgs well below 1 TeV and a lower bound on the scale of any new physics of several TeV.¹ If the SM is viewed as an effective theory with a cutoff about this scale, a confrontation with the naturalness problem of the weak scale is unavoidable, due to the quadratic divergences on the cutoff that affect the Higgs mass.² Clearly, the new physics must provide a mechanism to cut off these divergences, without altering the phenomenological success of the SM. In order to have a natural reductive effect, this in general requires partner particles associated with a new symmetry beyond the SM. Interestingly, these new particles are favored by the existence of dark matter in the universe as the SM cannot provide a viable candidate.

The most popular approach to solve such a naturalness/fine-tuning problem is weak scale supersymmetry implemented with R -parity,³ where the standard particles share the SM gauge symmetry with their supersymmetric partners. However a new problem comes forth, namely, the mass of some partner particles in the minimal supersymmetric extension of the SM implied by radiative corrections to the Higgs mass falls below the energy scale currently explored.¹ As none of these partners has been observed yet, the minimal version has to be expanded, for instance, with extra fields as in the next-to-minimal supersymmetric model.^{4,5}

Another approach contemplates an increase of the separation between the Higgs mass and the above cutoff based on the idea that the Higgs boson is light because it is in part a pseudo-Goldstone boson of a broken global symmetry. This possibility has been developed in the so-called little Higgs models with T -parity,^{6,7} twin Higgs models with twin symmetry,^{8–10} and mirror matter models with mirror symmetry,^{11,12} where the discrete symmetries relate the SM particles with their partners alleviating the naturalness problem of the weak scale. Replication of the SM particles is typical of these models beyond the SM.

While all these developments pursue a deep understanding of the electroweak gauge symmetry breaking and mass generation issues, they do not shed any light on the fermion family problem. These are related questions which should be answered by the same type of new physics beyond the SM. With so many families replicas, it is conceivable that the symmetry associated with the new partner particles and the symmetry associated with the existence of three fermion generations be related to each other in a unified description. In the end, they all are part of the same problem of family reproductions.

In this paper we argue that a symmetry indeed exists which accounts for the triplication of fermion families and the twin or mirror partners proposed to deal with the naturalness problem of the weak scale, then adding robustness to the twin and mirror symmetries. This symmetry is presymmetry,^{13,14} a predynamical symmetry hidden by the nontrivial topology of weak gauge fields which addresses the question of quark–lepton symmetry exhibited plainly in the electroweak sector of the SM when right-handed neutrinos are included. In Sec. 2 we discuss the main features of presymmetry and the rationale that sustains our approach to the solution of the family problem. In Sec. 3 we provide motivations to go beyond the SM with presymmetry, emphasizing the duplication of the SM particles with twin or mirror partners to have a residual presymmetry and so address the naturalness problem. The conclusion is given in Sec. 4.

2. Presymmetry

The quark–lepton symmetry has been extended from weak to electromagnetic interactions via a mechanism of charge fractionalization with topological attributes as in condensed matter physics.^{13,14} The approach is based on the charge symmetry between quarks and leptons. For an arbitrary weak hypercharge Y defined in terms

of the electric charge Q and the third component of weak isospin T_3 according to

$$Q = T_3 + a Y, \quad (1)$$

the quark–lepton charge symmetry is recognized from the following relation between hypercharges:¹⁴

$$\begin{aligned} Y(q_{L,R}) &= Y(\ell_{L,R}) - \frac{2}{3a} (3B - L)(\ell_{L,R}), \\ Y(\ell_{L,R}) &= Y(q_{L,R}) - \frac{2}{3a} (3B - L)(q_{L,R}), \end{aligned} \quad (2)$$

where $q_{L,R}$ and $\ell_{L,R}$ refer to quark and lepton weak partners within each of the three known families, right-handed neutrinos of $Y = 0$ included. Values of a used in the literature are $a = 1$ and $a = 1/2$. Although the value of the global part depends upon the hypercharge normalization, the charge symmetry is present anyway.

Presymmetry has to do with the exact correspondence between charge values of quarks and leptons if the global part was not present, clearly shown by Eq. (2). We set forth this connection in the electroweak quark–lepton symmetry principle: there is an intrinsic underlying equality of quark and lepton electroweak charges, regardless of how these charges are defined. We note that Majorana neutrinos are excluded from this symmetric picture of quarks and leptons which makes sense only for Dirac neutrinos.

2.1. Prequarks and preleptons

To discern the charge symmetry and the charge dequantization hidden in Eq. (2), we introduce the new primary states of prequarks and preleptons, denoted by \hat{q} and $\hat{\ell}$, which have the same quantum numbers of quarks and leptons, respectively, except hypercharge values. Hypercharges of prequarks and preleptons are the same as their lepton and quark weak partners. Specifically, we follow Eq. (2) to have¹⁴

$$\begin{aligned} Y(q_{L,R}) &= Y(\hat{q}_{L,R}) - \frac{2}{3a} (B - 3L)(\hat{q}_{L,R}), \\ Y(\ell_{L,R}) &= Y(\hat{\ell}_{L,R}) - \frac{2}{3a} (B - 3L)(\hat{\ell}_{L,R}), \end{aligned} \quad (3)$$

with prequark–lepton and prelepton–quark charge symmetries given by

$$\begin{aligned} Y(\hat{q}_{L,R}) &= Y(\ell_{L,R}), & Y(\hat{\ell}_{L,R}) &= Y(q_{L,R}), \\ (B - 3L)(\hat{q}_{L,R}) &= (3B - L)(\ell_{L,R}), & (B - 3L)(\hat{\ell}_{L,R}) &= (3B - L)(q_{L,R}), \end{aligned} \quad (4)$$

having $(B - 3L)(\hat{\ell}_{L,R}) = -(B - 3L)(\hat{q}_{L,R})$. In Eq. (3) the combination $B - 3L$ is in place of $3B - L$ because prequarks are the ones that now possess the lepton charges and preleptons the quark charges, as indicated in Eq. (4). From the latter we readily obtain $B(\hat{q}_{L,R}) = -1$ and $L(\hat{\ell}_{L,R}) = -1/3$; these values can be made positive if in Eqs. (3) and (4) we use $3L - B$ instead of $B - 3L$. Thus, $B - L$ and

not $B - 3L$ is the same for quarks and preleptons, as for prequarks and leptons. We see that the $B - 3L$ is essentially a bookkeeping global charge based on counting such that three preleptons make a system with one unit of $B - L$ charge, just as three quarks do.

We proceed with the charge symmetry exhibited in Eq. (2) and the charge dequantization described in Eq. (3) having B and L as ungauged global symmetries, quarks and leptons as the ultimate constituents of ordinary matter, and prequarks and preleptons as their basic bare states. The global, robust against local interactions, piece of hypercharge $2(B - 3L)/3a$ gets a topological significance associated with a topological charge or Pontryagin index which is independent of the normalization used for hypercharge. This implies a mixing of underlying local and topological charges, as discussed in the following.

2.2. *Topological quarks and presymmetry*

We now implement topologically the hidden quark–lepton charge symmetry shown above guided by the fact that any weak topological feature cannot have observable effects at the zero-temperature scale because of the smallness of the weak coupling. We then introduce the principle of weak topological charge confinement:¹⁴ observable particles have no weak topological charge, considering that the topological numbers are carried by the vacuum in which particles exist. It is secondary to that of gauge confinement, in the sense that electroweak forces by themselves cannot lead to confinement of topologically nontrivial particles. In the case of (pre)quarks, confinement is due to the strong color force.

This principle guarantees that quarks and leptons are topologically trivial and have no charge structure. Consequently, the charge structure held by Eqs. (3) and (4) does not apply to quarks, but to new entities that we name topological quarks. The assignments of topological quarks to the gauge groups of the SM, however, are the same of quarks. There is electroweak symmetry between topological quarks and preleptons, which may also be named topological leptons, as between prequarks and leptons (see Eq. (4)). Presymmetry is the statement of this charge symmetry. Analytically, it is the invariance of the bare electroweak Lagrangian under flavor transformations of a Z_2 group which interchange topological quarks (prequarks) and preleptons (leptons) weak partners, with no change on gauge and Higgs fields.

The feature that in baryons quarks are confined in threes containing each of the three colors requires at least a weak topological charge associated with a bookkeeping Z_3 charge, defining a nontrivial value $+1$ for topological quarks to have the equivalence between three topological quarks and three quarks. Hence, the 3 of this modulo charge in topological quarks, based on counting, is due to the number of colors. By presymmetry, preleptons also have a Z_3 charge equal to $+1$. We argue below that the 3 of this modulo charge in preleptons is due to the number of families. When the bookkeeping charge is 3, the set has no topological charge and trivial topology, although leptons do not confine.

2.3. Topological charge and charge normalization

It is assumed that prequarks and preleptons interact with the standard gauge and Higgs fields through a Lagrangian like that of the SM with quarks and leptons excepting hypercharge couplings and incorporation of Dirac right-handed neutrinos. The cancellation of gauge anomalies generated by their nonstandard charges leads to the appearance of a topological charge. More specifically, in the presymmetric scenario of prequarks and leptons, each prequark changes its hypercharge by the same value, a charge shift which can be written as follows:¹⁴

$$Y(\hat{q}) \rightarrow Y(\hat{q}) + \frac{n}{6a} Q^{(3)}(\hat{q}) = Y(\hat{q}) - \frac{n}{6a} (B - 3L)(\hat{q}), \quad (5)$$

where n is the topological charge of a $SU(2)_L$ instanton, with a Z_3 counting number $Q^{(3)}$ attached to it equal to ± 1 for nontrivial topology and 0 for trivial topology, just as if the topological charge were itself a Z_3 charge. It is due to the hypothesis of the approach. We define $Q^{(3)} = -(B - 3L)$ in accordance with Eq. (3), noting that three prequarks pass to a system of $Q^{(3)} = 3 (= 0)$, the neutral element of Z_3 .

The required value for the topological index is $n = 4$ because of the gauge anomaly cancellation, demanded for gauge invariance and renormalizability, and so consistency of the gauge theory. All assignments of hypercharge (times the conventional parameter a) of topological quarks are determined, resulting in the hypercharge of quarks and their observed electric charges. The value singled out for the topological charge does not depend on a , as indicated in Subsec. 2.1. Now, in order to get a lower value one has to go beyond the SM. For instance, a topological charge $n = 2$ is obtained by a symmetric duplication of the $SU(2)_L$ gauge group as in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ left-right symmetric models, which we will report on in a separate paper.

The first term of the right-hand side of Eq. (5) is associated with a local charge created by local fields and the second with a topological charge related to a weak instanton which cannot be generated by local operations. We then have here a concrete example of a mechanism of charge fractionalization in which states of local fields — prequarks — pass to states with a topological character — topological quarks — which allows local and topological charges to be mixed.¹⁵ It is a hidden charge structure that explains the fractional charge of topological quarks as in condensed matter physics.

It is concluded that topological quarks are involved in a vacuum gauge field configuration of winding number $n_W = 4$, if we use gauge freedom to set $n_W = 0$ for the one containing prequarks. The transformation of prequarks into topological quarks is via an Euclidean topological weak-instanton with topological charge $n = 4$, interpreted in Minkowski spacetime as a quantum-mechanical tunneling event between vacuum states of weak $SU(2)_L$ gauge fields with different topological winding quantum numbers. In this sense, prequarks and topological quarks reside in different vacua. In other words, the difference between topological quarks and prequarks is

the nonequivalence between the topological vacua of their weak gauge configurations, tunneled by a weak four-instanton which carries the topological charge and induces the universal fractional piece of charge needed for normalization. Similarly, the passage from topological quarks to quarks of same hypercharge values is via a $n = -4$ vacuum tunneling weak-instanton, when three confined topological quarks of neutral Z_3 total charge pass to three confined quarks of trivial topology.

We remark that the charge normalization and the zero weak topological charge in quarks and hadrons effectively remove the extremely large time scale for the transitions from prequarks to topological quarks and from the latter to quarks, associated with the extreme smallness of the instanton transition probability. Besides, these transitions do not happen in the real world because, as argued in Sec. 3, prequarks and topological quarks are not real dynamical entities.

Gauge anomaly cancellation and charge normalization in the presymmetric scenario of topological quarks and preleptons are done in a similar way.¹⁴ The topological structure and charge dequantization in preleptons, which are symmetric to the ones in topological quarks, are annulled by the four-instanton effect, leading to leptons with trivial topology and charge as in prequarks (see Eqs. (3) and (4)).

2.4. Number of families

At the level of quarks and leptons, the number 3 in Eq. (2), which is also the order of the additive group Z_3 associated with the topological charge in the framework of topological quarks, goes with the color number because of the correlation in quarks between the baryon number and the number of colors. Indeed, the occurrence of three colors accounts for the fact that baryons made of three quarks of baryon number $B = 1/3$, have $B = 1$; the three quarks containing each of the three possible colors of color charge. This can be accommodated in the relation $B = 1/N_c$, in the case of quarks. But, in the hidden scenario of prequarks and preleptons, the 3 in Eq. (3) cannot be conceived in the same way by the facts that $B = -1$ for prequarks and that color is not a prelepton quantum number. The number 3 at the two levels of descriptions given in Eqs. (2) and (3) therefore has to be interpreted differently. At the hidden level, it must be associated with a numerable property of prequarks and preleptons and the number of families N_f is the only other available degree of freedom. Thus, whereas the partition of topological charges in the scenario with topological quarks depends on the number of colors, the partition in the scenario with preleptons has to be in conformity with the number of families; prequarks and leptons are topologically trivial. To relate the 3 in Eq. (3), which is the order of the additive group Z_3 for the topological charge in colorless preleptons, with the number of families becomes inevitable if we assume that this number in Z_3 must be explained by physics of the SM.¹⁴ No other new physics is needed to understand that number 3. It would be surprising to not have such a connection, considering the fact that the SM offers no reason for the triplication of families.

In the end, preleptons have a lepton number marked by the number of families: $L = -1/N_f$, as inferred from Subsec. 2.1. Thus, for presymmetric prequarks and leptons one has $B - L = -1$, whereas for preleptons and topological quarks presymmetry leads to $B - L = 1/3$, with $N_f = N_c = 3$. Now, the prequark hypercharge shift in Eq. (5) can be written as $\Delta Y = 2/aN_f$, displaying the expected meaning of the 3 in the hypercharge relationships of Eq. (3).

The solution of the family problem is given by presymmetry which demands the same number of families of quarks and leptons and the exact correspondence between this number and that of quark colors.

3. On a Residual Presymmetry

Electroweak presymmetry is hidden at the level of standard quarks and leptons. Fractional charge is generated in a peculiar manner but only mathematically. Neither topological quarks, prequarks and preleptons are real dynamical entities with definite mass values nor the associated presymmetry has a mass scale breaking. They are not the particles that do the job with the physical gauge and Higgs fields of the SM. All of them are bare prestates of quarks and leptons which are seen as convenient mathematical entities out of which the actual particle states are built up. It is meant as a scheme that guesses at a new hidden charge symmetry, presymmetry, which embraces quarks and leptons. If taken as a real dynamical model, it possesses serious problems for presymmetric topological quarks, prequarks and preleptons cannot be physical states; in simple terms, these do not exist. For instance, transitions from prequarks to topological quarks and from the latter to quarks would be faced badly with the negligible smallness of the instanton transition probability if the former were real objects, but they are not. This is what allows to define the mixing of local and topological charges in Eq. (5).

In spite of that, the proposal provides a theoretical framework which has many physical implications:¹⁴ it explains the fractional charge of quarks and the quark-lepton charge relations; it states that the number of fermion generations has to be equal to the number of quark colors; it predicts $B - L$ conservation and the Dirac character of massive neutrinos; it accounts for the topological charge conservation in quantum flavor dynamics; it explains charge quantization and the no observation of fractionally charged hadrons.

Even so, there is nothing physically new and nothing has been altered at the level of the SM. This is disturbing, because one may expect some other new physics to account for the above implications and therefore ask for the Occam's razor: "Entities should not be multiplied unnecessary." To avoid it, a residual presymmetry in the sense of Ekstein^{16,17} has to be generated. Besides, it is really difficult to accept that the hidden picture of the discrete presymmetry cannot be tested; if this is the case, it is impossible to either verify or falsify the proposal. These are strong motivations to take presymmetry beyond the SM. Other reasons are to extend presymmetry from matter to forces and from the electroweak to the strong sector,

i.e. to have presymmetry for the full Lagrangian of fundamental interactions, then acquiring more significance with a strong influence on the course of the new physics. A residual presymmetry based on these motivations requires a doubling of the SM particles, whose existence will make the substantiation of the proposition by leading to new experimentally observable predictions. Due to the connection between the number of fermion generations and the number of quark colors, the new families must be nonsequential, duplicating the gauge groups. On the other hand, due to its topological character, presymmetry is unrelated to the energy scale and appears to be transverse to everything, prompting in particular its enlargement to the forces of the SM independently of the ultraviolet completion of the theory.

3.1. *Twin symmetry from presymmetry*

The simplest duplication of the SM keeps spin and handedness. It is a plain copy of the SM particles much as the second and third generations of quarks and leptons are mere copies of the first generation; copies all that can be regarded as implied by presymmetry. Now we describe how under this replication of particles a residual presymmetry comes out and extends from matter to forces and from weak to strong interactions.

On the one hand, it is the hidden charge symmetry relating quark and lepton multiplets, as explained in Sec. 2, and their respective partners denoted by tildes:

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\nu_L, e_L), & u_R &\leftrightarrow \nu_R, & d_R &\leftrightarrow e_R, \\ (\tilde{u}_L, \tilde{d}_L) &\leftrightarrow (\tilde{\nu}_L, \tilde{e}_L), & \tilde{u}_R &\leftrightarrow \tilde{\nu}_R, & \tilde{d}_R &\leftrightarrow \tilde{e}_R, \end{aligned} \quad (6)$$

where right-handed neutrinos have been included. The underlying presymmetry between fermions is hidden by the charge shifts induced by the topological charges associated with the configurations of weak gauge fields. Gauge and Higgs fields are not changed by the presymmetric interchanges that leave invariant the electroweak part of the bare Lagrangian.

On the other hand, there is a similar hidden charge symmetry between quarks and the partners of leptons, and between their copies, respectively:

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\tilde{\nu}_L, \tilde{e}_L), & u_R &\leftrightarrow \tilde{\nu}_R, & d_R &\leftrightarrow \tilde{e}_R, \\ (\tilde{u}_L, \tilde{d}_L) &\leftrightarrow (\nu_L, e_L), & \tilde{u}_R &\leftrightarrow \nu_R, & \tilde{d}_R &\leftrightarrow e_R. \end{aligned} \quad (7)$$

Here the electroweak symmetry which interchanges the gauge and Higgs bosons with their partners requires that the corresponding coupling constants be equal.

The Z_2 symmetries of Eqs. (6) and (7) lead to the following one between quarks and their partners, and between leptons and their duplicates:

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\tilde{u}_L, \tilde{d}_L), & u_R &\leftrightarrow \tilde{u}_R, & d_R &\leftrightarrow \tilde{d}_R, \\ (\nu_L, e_L) &\leftrightarrow (\tilde{\nu}_L, \tilde{e}_L), & \nu_R &\leftrightarrow \tilde{\nu}_R, & e_R &\leftrightarrow \tilde{e}_R. \end{aligned} \quad (8)$$

Besides, there is symmetry between electroweak gauge and Higgs bosons and their partners, with same coupling constants. This Z_2 symmetry, but not the others, remains exact after the underlying charge normalization on fermions. Moreover, it extends to strong interactions for equal gauge couplings of the two color groups. In this case, an observable residual Z_2 symmetry exists, the required residual presymmetry, which includes the strong sector, relates every particle of the SM with its partner particle and constrains the corresponding coupling constants to be equal, just as in twin matter models with twin symmetry. As a consequence, the existence of two symmetric Higgs doublets alleviates the hierarchy problem.^{8–10} This is discussed in the following.

Under a full duplication of the SM, there are two renormalizable couplings between particles of the SM and their partners allowed by gauge invariance: $\lambda\phi^\dagger\phi\tilde{\phi}^\dagger\tilde{\phi}$ and $\epsilon B^{\mu\nu}\tilde{B}_{\mu\nu}$, where $\phi, \tilde{\phi}$ are the Higgs doublets of the SM and its copy respectively and $B^{\mu\nu}, \tilde{B}^{\mu\nu}$ are the hypercharge field strength tensors. In reference to the Higgs sector, there is a limit in which the Higgs scalar may be treated as a pseudo-Goldstone boson. To see it, the Higgs potential is written as

$$V = -\mu^2(\phi^\dagger\phi + \tilde{\phi}^\dagger\tilde{\phi}) + \lambda(\phi^\dagger\phi + \tilde{\phi}^\dagger\tilde{\phi})^2 + \delta[(\phi^\dagger\phi)^2 + (\tilde{\phi}^\dagger\tilde{\phi})^2], \quad (9)$$

where the term proportional to λ contains the above coupling of Higgs-doublet partners. The potential maintains a $U(4)$ global symmetry in the limit $\delta \rightarrow 0$. The model presents two nontrivial vacua which rely on whether $\delta > 0$ (symmetric vacuum) or $\delta < 0$ (asymmetric vacuum).¹⁰ The symmetric vacuum, where both Higgs doublets get the same vacuum expectation values, is the interesting case. Here, $\langle\phi\rangle = \langle\tilde{\phi}\rangle = v$ with $v^2 = \mu^2/(4\lambda + 2\delta)$. Although gauge and Yukawa couplings violate the global symmetry, the discrete symmetry that interchanges particles and partners is respected.

In the SM, the most significant quadratically divergent one-loop contributions to the Higgs potential involve the top quark, the gauge bosons, and the Higgs scalar. Keeping just the one-loop top quark correction,

$$\mu^2 = \mu_o^2 + a_t\Lambda_t^2, \quad (10)$$

where μ_o is the bare parameter, $a_t = 3\lambda_t^2/8\pi^2$, $\lambda_t = m_t/v_t \sim 1$ is the top quark Yukawa coupling constant, and Λ_t is the cutoff from new physics. In the extended model with Z_2 symmetric partners, the quadratic divergence maintains its form in both sectors and so the $U(4)$ symmetry. The spontaneous symmetry breaking $U(4) \rightarrow U(3)$ in the limit $\delta \rightarrow 0$ leads to one massless Higgs boson, as expected. Corrections to the Higgs quartic interactions that are not invariant under the global symmetry, such as the δ term in Eq. (9), provide mass to the Higgs of order the weak scale.^{8,9}

The quadratic divergence in μ^2 from the top quarks is alleviated in the duplicated model. In fact, a measure of fine-tuning is

$$\left(\frac{\delta\mu^2}{\mu^2}\right)_t = \frac{a_t\Lambda_t^2}{\mu^2}, \quad (11)$$

with $\mu^2 = m_h^2/2 = 2v^2\lambda$ in the SM and $\mu^2 = m_+^2/2$ in the duplicated model, where $m_+^2 = 4v^2(2\lambda + \delta)$ is the mass of the heavier physical Higgs boson and $m_-^2 = 4v^2\delta$ is that of the lighter. In the SM, the bound from precision electroweak measurements is $m_h < m_{EW} \approx 186$ GeV.¹ In the duplicated model, the bound is $m_+ m_- < m_{EW}^2$. Thus, because of a large m_+ , the fine-tuning in the μ^2 parameter due to the top quark is alleviated.

More quantitatively, the scale of new physics depends on the amount of fine-tuning that is allowed. A model is considered ideal if $|\delta\mu^2/\mu^2| \lesssim 5$, corresponding to no significant electroweak fine-tuning.¹⁸ At the experimental limit of about 114 GeV for m_h in the SM and m_- in the duplicated model, and taking m_+ to the largest value consistent with electroweak precision tests, the ideal value of the upper limit on Λ_t is

$$\Lambda_t = \frac{2\pi}{\sqrt{3}\lambda_t} m_h \left| \frac{\delta\mu^2}{\mu^2} \right|_t^{1/2} \sim 0.9 \text{ TeV} \quad (12)$$

in the SM and

$$\Lambda_t = \frac{2\pi}{\sqrt{3}\lambda_t} m_+ \left| \frac{\delta\mu^2}{\mu^2} \right|_t^{1/2} \sim 2.5 \text{ TeV} \quad (13)$$

in the duplicated model, which shows the improvement of naturalness of the Higgs sector. This cutoff can be scaled up by allowing a moderate fine-tuning.

Regarding the one-loop Higgs correction to the quadratic divergences in the SM, we have

$$\mu^2 = \mu_o^2 - a_H \Lambda_H^2, \quad (14)$$

where $a_H = 3\lambda/8\pi^2$ with λ being the quartic coupling constant of the Higgs potential and Λ_H the cutoff from new physics for this divergence. It leads to the result

$$\left(\frac{\delta\mu^2}{\mu^2} \right)_H = - \frac{a_H \Lambda_H^2}{\mu^2}. \quad (15)$$

In the duplicated model with Z_2 symmetry the correction goes with $a_H = (5\lambda + 3\delta)/8\pi^2$. The values for the upper bound on the cutoff are

$$\Lambda_H = \frac{4\pi}{\sqrt{3}} v \left| \frac{\delta\mu^2}{\mu^2} \right|_H^{1/2} \sim 2.8 \text{ TeV} \quad (16)$$

in the SM and

$$\Lambda_H = \frac{4\pi\sqrt{2}}{\sqrt{5+\gamma}} v \left| \frac{\delta\mu^2}{\mu^2} \right|_H^{1/2} \sim 3.0 \text{ TeV} \quad (17)$$

with $\gamma = m_-^2/m_+^2$ in the extended model. Here only a little increase in the scale of new physics relative to the SM is feasible. In the case of gauge boson loops, contributions to quadratic divergences have the same form as from Higgs bosons and also become smaller in magnitude compared with that from top quark.

Thus, Λ_t sets the scale of new physics in this domain of parameters where a relatively light Higgs, as favored by precision electroweak data, is assumed. Since $\Lambda_t \sim 0.9$ TeV of the SM is within reach of LHC, manifestations of the new physics are expected. In the case of the duplicated model, if $\Lambda_t \sim 2.5$ TeV were outside reach of LHC, this extended model with no other new physics would be perfectly natural. There would be consistency with any bound from precision electroweak measurements because partner particles are neutral with respect to the SM gauge interactions. If the Higgs boson does not exist and other symmetry breaking mechanism is operative, the duplication of the SM goes anyway.

3.2. Mirror symmetry from presymmetry

There is an alternative, also simple copy of the SM leading to a residual presymmetry. It is the mirror-symmetric case where an exact parity symmetry is claimed as due. Left-handed weak gauge bosons act on SM particles and right-handed ones on their partners. Now, instead of Eq. (6), the hidden Z_2 symmetry is according to

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\nu_L, e_L), & u_R &\leftrightarrow \nu_R, & d_R &\leftrightarrow e_R, \\ (\tilde{u}_R, \tilde{d}_R) &\leftrightarrow (\tilde{\nu}_R, \tilde{e}_R), & \tilde{u}_L &\leftrightarrow \tilde{\nu}_L, & \tilde{d}_L &\leftrightarrow \tilde{e}_L. \end{aligned} \quad (18)$$

In place of the charge symmetry in Eq. (7), we have

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\tilde{\nu}_R, \tilde{e}_R), & u_R &\leftrightarrow \tilde{\nu}_L, & d_R &\leftrightarrow \tilde{e}_L, \\ (\tilde{u}_R, \tilde{d}_R) &\leftrightarrow (\nu_L, e_L), & \tilde{u}_L &\leftrightarrow \nu_R, & \tilde{d}_L &\leftrightarrow e_R. \end{aligned} \quad (19)$$

Finally, instead of Eq. (8), we obtain mirror symmetry

$$\begin{aligned} (u_L, d_L) &\leftrightarrow (\tilde{u}_R, \tilde{d}_R), & u_R &\leftrightarrow \tilde{u}_L, & d_R &\leftrightarrow \tilde{d}_L, \\ (\nu_L, e_L) &\leftrightarrow (\tilde{\nu}_R, \tilde{e}_R), & \nu_R &\leftrightarrow \tilde{\nu}_L, & e_R &\leftrightarrow \tilde{e}_L. \end{aligned} \quad (20)$$

All gauge coupling constants in both sectors are also related by mirror parity. The Higgs sector is as in the above model, alleviating the hierarchy problem in the same way, as in mirror matter models with mirror symmetry.¹¹ Thus, the residual presymmetry that is demanded can be seen as being the cause of twin symmetry or mirror symmetry. It is worth noting that these symmetries emerge independently of the solution of the quadratic divergence problem.

Presymmetry remains hidden and the model is in trouble if there is no copy of the SM particles. Majorana neutrinos and sequential families, such as a fourth generation, also bring problems to the idea of presymmetry which requires equal numbers of fermion families and quark colors. If anything of this could occur, one would go back to the starting point of the model and state that the quark-lepton charge symmetry which supports presymmetry is just an accidental interplay of quantum numbers, which is really hard to be accepted.

4. Conclusion

Presymmetry is a hidden electroweak symmetry of the SM that embraces quarks and leptons with many physical implications. It explains the fractional charge of quarks and the quark–lepton charge relations. It states that the number of fermion generations has to be equal to the number of quark colors. It predicts $B - L$ conservation and the Dirac character of massive neutrinos. It accounts for the topological charge conservation in quantum flavor dynamics. It explains charge quantization and the no observation of fractionally charged hadrons.

In order to be substantiated, however, it must be extended beyond the SM as to have a residual presymmetry that includes partner particles and the strong sector. And the case has been made in which a close relation exists between this residual presymmetry and the twin and mirror symmetries proposed in the literature to mitigate the naturalness problem of the weak scale. The duplication of fermion families and gauge bosons of the SM is predicted by presymmetry independently of the ultraviolet completion of the theory, even if the Higgs particle is discarded by experiment, which adds robustness to twin and mirror symmetries. The established connection is so strong that the search for twin or mirror symmetry becomes the possible test of presymmetry. Experimentally observable predictions are extracted from twin or mirror matter models.^{10,19,20} If the physics beyond the SM repairs its left–right asymmetry, mirror symmetry should be the one realized in nature.

Acknowledgments

We would like to thank J. Gamboa for helpful comments. This work was supported by the Departamento de Investigaciones Científicas y Tecnológicas, Universidad de Santiago de Chile, Usach.

References

1. Particle Data Group (K. Nakamura *et al.*), *J. Phys. G* **37**, 075021 (2010).
2. R. Barbieri and A. Strumia, arXiv:hep-ph/0007265.
3. D. J. H. Chung *et al.*, *Phys. Rep.* **407**, 1 (2005).
4. M. Bastero-Gil *et al.*, *Phys. Lett. B* **489**, 359 (2000).
5. U. Ellwanger, C. Hugonie and A. M. Teixeira, arXiv:0910.1785.
6. H. C. Cheng and I. Low, *J. High Energy Phys.* **0309**, 051 (2003).
7. H. C. Cheng and I. Low, *J. High Energy Phys.* **0408**, 061 (2004).
8. Z. Chacko, H. S. Goh and R. Harnik, *Phys. Rev. Lett.* **96**, 231802 (2006).
9. Z. Chacko, Y. Nomura, M. Papucci and G. Perez, *J. High Energy Phys.* **0601**, 126 (2006).
10. R. Barbieri, T. Gregoire and L. J. Hall, arXiv:hep-ph/0509242.
11. R. Foot and R. R. Volkas, *Phys. Lett. B* **645**, 75 (2007).
12. R. Foot, H. Lew and R. R. Volkas, *Phys. Lett. B* **272**, 67 (1991).
13. E. A. Matute, *Phys. Lett. B* **538**, 66 (2002).
14. E. A. Matute, *Int. J. Mod. Phys. A* **22**, 3669 (2007).

15. F. Wilczek, arXiv:cond-mat/0206122.
16. H. Ekstein, *Phys. Rev.* **153**, 1397 (1967).
17. H. Ekstein, *Phys. Rev.* **184**, 1315 (1969).
18. J. F. Gunion, arXiv:0804.4460.
19. A. Y. Ignatiev and R. R. Volkas, *Phys. Lett. B* **487**, 294 (2000).
20. R. Schabinger and J. D. Wells, *Phys. Rev. D* **72**, 093007 (2005).